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EXAMINATION OF VIBRATION OF THE WOODEN BEAMS UNDER LOAD

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Present paper concerns the problem of vibration of the wooden beams under load. Effect of the tensile, compressive and bending stresses on the vibration damping of pine-wood, spruce-wood and hornbeam-wood beams have been investigated. Also, there has been experimentally confirmed the relationships, known in the theory, between the tensile and compressive forces and the natural frequency of the beams loaded by those forces.

1. Introduction

Efforts have been made during the work to find the influence of different states of stresses on the transverse vibration of the beams. Variations of the stress states are accompanied by changes of vibration form, frequency and vibration damping. Vibration forms are related to the eigenfunctions, which in the simplest case (sinusoidal vibration) are characterized by the division of the whole length of the beam on the vibrating elements what causes the increase of nodes number. This rule may be generalized.

The authors were trying to

1. confirm experimentally the known relationship of the natural frequency of beam vibration under tensile and compressive forces, and
2. represent the relationship between logarithmic decrement and tensile, compressive and bending stresses of the beam.

2. Effect of stress states on beam vibration

Considering a beam as the Clapeyron's system (linear-elastic) the following integral equation may be written, describing the motion of the beam, Pisarenko, Yakovlev and Matveev (1976)

$$w(x,t) + \int_0^l R^*(x,\xi)\frac{\partial^2 w(\xi,t)}{\partial t^2}m(\xi)d\xi =$$
$$= \int_0^l R^*(x,\xi)\alpha\xi\frac{\partial w(\xi,t)}{\partial t}d\xi + \int_0^l R^*(x,\xi)z(\xi)d\xi \tag{2.1}$$

where

$R^*(x,\xi)$	–	influence function,
$w(x,t)$	–	lateral displacement of the beam in point x,
$m(\xi)$	–	mass per length unit,
$\alpha(\xi)$	–	damping coefficient,
$z(\xi)$	–	exciting force of vibration,
t	–	time

Solution of Eq (2.1) is expected in the form

$$w(x,t) = \frac{1}{\sqrt{m(x)}}\sum_{i=1}^{\infty}\psi_i(x)T_i(t) \tag{2.2}$$

where $\psi_i(x)$ stand for the eigenfunctions. Assuming that

$$R(x,\xi) = R^*(x,\xi)\sqrt{m(x)m(\xi)}$$

and remembering that the kernel $R(x,\xi)$ may be expanded in eigenfunction according to the formula

$$R(x,\xi) = \sum_{i=1}^{\infty}\frac{\psi_i(x)\psi_i(\xi)}{k_i^2}$$

where k_i are the natural frequencies of vibration.

Assuming also that

$$\int_0^l \psi_i(\xi)\psi_n(\xi)\frac{\alpha(\xi)}{m(\xi)}d\xi = \begin{cases} 0 & \text{when} \quad i \neq n \\ 2h_i & \text{when} \quad i = n \end{cases} \tag{2.3}$$

and considering Eq (2.2) into Eq (2.1) we have

$$\sum_{i=1}^{\infty}\psi_i(x)\Big\{T_i(t) + \frac{1}{k_i^2}\Big[\ddot{T}_i(t) + 2h_i\dot{T}_i(t) - a_i(t)\Big]\Big\} = 0$$

Because of the eigenfunctions orthogonality $\psi_i(x)$ one gets

$$\ddot{T}_i(t) + 2h_i\dot{T}_i(t) + k_i^2 T_i(t) = a_i(t) \tag{2.4}$$

where

$$a_i(t) = \int_0^l \psi_i(\xi)\frac{z(\xi,t)}{\sqrt{m(\xi)}}d\xi$$

Eq (2.4) is solved by using the Fourier transformation

$$-\omega^2\bar{T}_i(\omega) + 2h_i j\omega\bar{T}_i(\omega) + k_i^2\bar{T}_i(\omega) = \bar{a}_i(\omega)$$

therefore

$$\bar{T}_i(\omega) = \bar{G}_i(\omega)\bar{a}_i(\omega)$$

where

$$\bar{G}_i(\omega) = \frac{1}{k_i^2 - \omega^2 + 2h_i j\omega} \qquad j = \sqrt{-1} \tag{2.5}$$

Modulus $|G_i(\omega)|$ is the amplitude characteristic

$$|G_i(\omega)| = A_i(\omega) = \frac{1}{\sqrt{(k_i^2 - \omega^2)^2 + 4h_i^2 j\omega}} \tag{2.6}$$

Let us consider differential equation describing vibration of the beam to determine its natural frequency

$$EJ\frac{\partial^4 w}{\partial x^4} + S\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{2.7}$$

where
- EJ – flexural rigidity,
- S – force which is axially compressing the beam,
- m – mass per length unit.

Seeking for the solution in the form

$$W(x,t) = w(x)\sin(\omega t + \varphi) \tag{2.8}$$

and after substituting Eq (2.8) into Eq (2.7) one gets the ordinary differential equation

$$EJ\frac{d^4 w(x)}{dx^4} + S\frac{d^2 w(x)}{dx^2} + \omega^2 m w(x) = 0 \tag{2.9}$$

The solution to Eq (2.9) satisfying the boundary conditions on the left end of the beam $w(0) = w'(0) = 0$, has the following form

$$w(x) = C\Big(\frac{\sin a_1 x}{a_1} - \frac{\sinh a_2 x}{a_2}\Big) + D(\cos a_1 x - \cosh a_2 x) \tag{2.10}$$

where

$$a_1^2 = \alpha^2 + \sqrt{\alpha^4 + \beta^2} \qquad\qquad a_2^2 = -\alpha^2 + \sqrt{\alpha^4 + \beta^2}$$

$$\alpha^2 = \frac{S}{2EJ} \qquad\qquad \beta^2 = \omega^2 \frac{m}{EJ} \qquad\qquad C, D - \text{constant}$$

Taking into account the boundary conditions $w(l) = w'(l) = 0$ we get

$$\begin{aligned} &C\Big(\frac{\sin a_1 l}{a_1} - \frac{\sinh a_2 l}{a_2}\Big) + D(\cos a_1 l - \cosh a_2 l) = 0 \\ &C(\cos a_1 l - \cosh a_2 l) + D(-a_1 \sin a_1 l - a_2 \sinh a_2 l) = 0 \end{aligned} \tag{2.11}$$

The nontrivial solution exists, when the characteristics determinant of Eq (2.11) is equal to zero. On this base we can determine the natural frequencies of vibration, from the equation

$$2(1 - \cosh a_2 l \cos a_1 l) + \Big(\frac{a_2}{a_1} - \frac{a_1}{a_2}\Big) \sin a_1 l \sinh a_2 l = 0 \tag{2.12}$$

Amplitude characteristic (2.6) contains the term h_i. In our investigation we would like to determine how this relationship depends on the axial force S. Assuming that $\alpha(\xi) =$ constant and $m(\xi) =$ constant then on the grounds of Eq (2.3) we have $h_i = h =$ constant. On account of these assumptions it is sufficient for our purposes to examine the damping of the fundamental vibration.

3. Investigation of the straight wooden beams axially stretched vibration, compressed and bent

Application of wood as damping material and manufacturing wooden sound boards in musical instruments produce requirements for determination of an effect of the wooden beam load on its damping properties.

The authors aimed at wood damping properties presentation in form of the characteristics (3 classes of wood) subject to the specified stresses. Furthermore, the purpose of this work is to describe dependence of the natural frequency of the wooden specimens on the load increment of the tensile and compressive type.

3.1. Method of the measurement

Measurements have been carried out using Brüel & Kjær 2515 vibration analyzer supplied with piezoelectric accelerometer 4381. Values of the tensile forces

S are changing within the range from 0 to 35 kG in 2.5 kG steps and compressive forces S from 0 to 20 kG in 2.5 kG steps. Bending is realized by introducing the bending moments M in the beam supports (Fig.1).

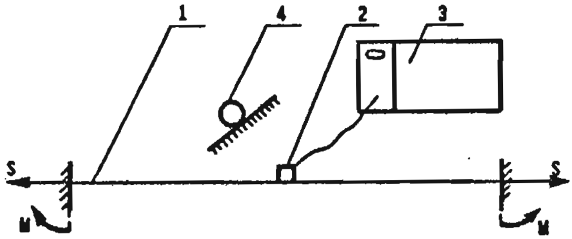

Fig. 1. 1 – the beam under consideration, 2 – accelerometer, 3 – vibration analyzer, 4 – ball forcing the vibration

The vibration forcings are realized by impulse action of the steel ball, which falling down on the beam from the specified height and strikes the beam.During examination the repeatability of the forcings have been assured. Three wooden beams (well seasoned) of the rectangular cross-section have been examined.

They are

- pine-wood beam of the size 500 mm × 40 mm × 5 mm
- spruce-wood beam of the size 500 mm × 40 mm × 3.5 mm
- hornbeam-wood beam of the size 500 mm × 60 mm × 3 mm.

For determination of the vibration decay, the average values of the logarithmic decrement have been assumed for the m successive semi-periods

$$\delta = \frac{1}{m} \ln \frac{a_n}{a_{n+m}} \tag{3.1}$$

4. Examination results

4.1. Vibration frequency

For illustration of frequency changes in the beam vibrations with increase of the tensile force and of the compressive force – the characteristics have been plotted on which theoretical curves of the changes is also marked (Fig.2).

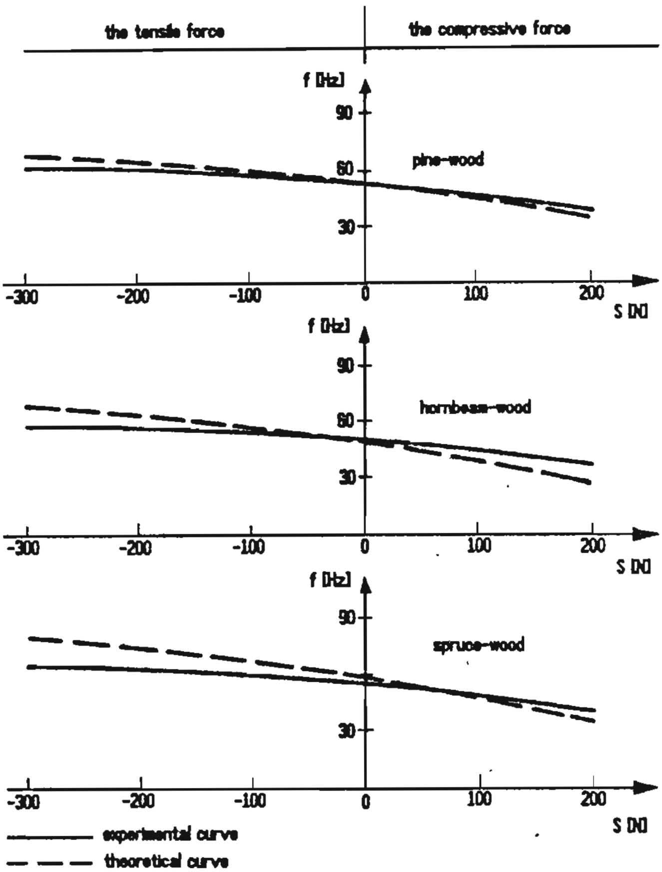

Fig. 2.

4.2. Vibration damping in the wood

For determination of vibration damping of the pine-woods, spruce-wood and hornbeam-wood beams when loading them by tensile and compressive forces and bending moments, the following diagrams have been prepared
— logarithmic decrement dependence on the tensile force and the compressive force (Fig.3)
— logarithmic decrement dependence on the deflection (Fig.4).

On the grounds of the experiments carried out one can state as follows.

- From the comparison of the known formula on vibration frequency of the beam loaded by an axial force with the experimental results (Fig.2) it is

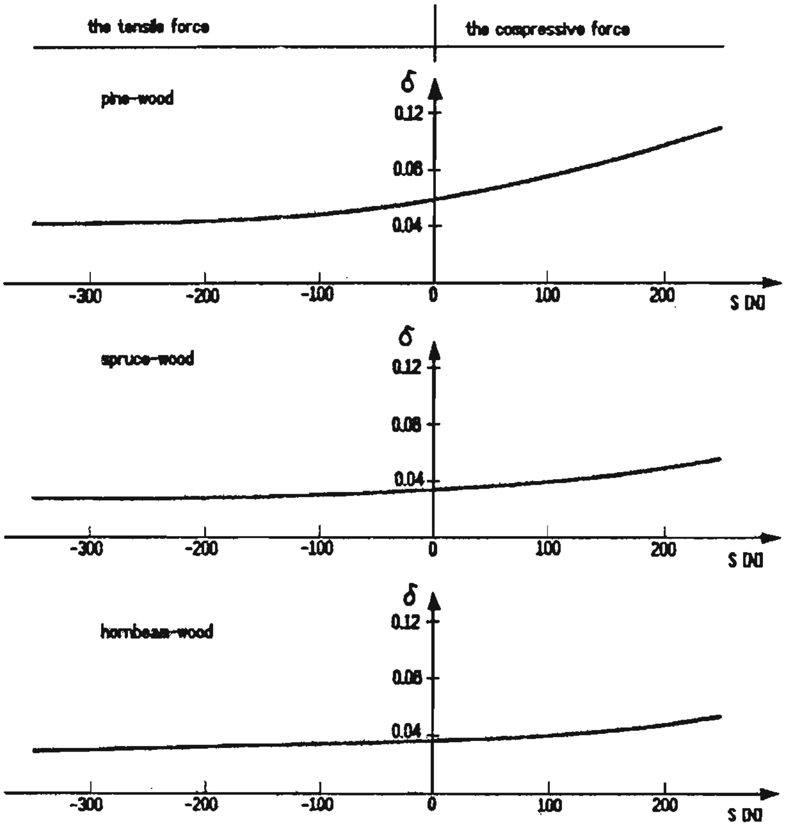
the tensile force
the compressive force
pine-wood
spruce-wood
hornbeam-wood
δ
0.12
0.08
0.04
-300
-200
-100
0
100
200
S [N]

Fig. 3.

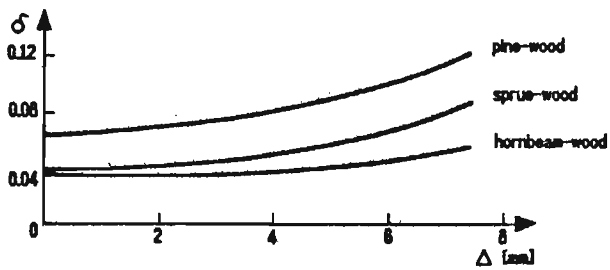
δ
0.12
0.08
0.04
0
2
4
6
8
Δ [mm]
pine-wood
sprue-wood
hornbeam-wood

Fig. 4.

seen that the rate of frequency increase is smaller than that resulting from this formula. Similarly, the rate of frequency decrease with increment of the compressive force is smaller than the theoretical one.

- Dependence of the logarithmic decrement from axial force S have been presented in diagrams (Fig.3). From the course of curves it is seen that decrement is increasing together with the compressive force of the beam. Similarly, decrement is increasing while bending the beam (Fig.4). On the other hand, in the case of tension it is retained, on the average, on the same level and even decreasing slightly (Fig.3).

The results may be used in different damping structures or for technological needs of wooden sound board manufacture in the string instruments.

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Badanie drgań belek drewnianych znajdujących się pod obciążeniem

Streszczenie

Praca dotyczy zagadnienia drgań belek drewnianych obciążonych. Zbadano wpływ naprężeń rozciągających, ściskających oraz zginających na tłumienie belki sosnowej, świerkowej oraz grabowej. Potwierdzono również doświadczalnie znane z teorii zależności pomiędzy siłami rozciągającymi i ściskającymi, a częstotliwością belek obciążonych tymi siłami.

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